# BARYON MASSES FROM QCD CURRENT CORRELATORS AT $T \neq 0$

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### Abstract

Correlation functions of QCD currents with quantum numbers of nucleon and $\Delta$-isobar are considered at finite temperatures. Corrections of order $T^4$ to the correlators are calculated and interpreted in terms of thermal mass shifts using a QCD sum rules type of argument. In both cases the masses decrease with $T$.

Key-Words : correlators, finite temperature, QCD sum rules, baryons

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The study of QCD current correlators at finite temperatures has been of increased interest recently (for an extensive review, see Ref.[1]). By studying their $T$-dependence one may get an insight on the changes in the hadron spectrum as the temperature approaches the point $T_c \approx 150$ MeV at which the hadron phase is expected to undergo chiral and deconfining phase transitions. One may try to extend the method of operator product expansion (OPE) from $T = 0$ to the case of $T \neq 0$ and use it to evaluate the finite temperature correlators. The $T$-dependence of non-perturbative condensates is due to interactions with the heat bath. Since at low $T$ the heat bath is dominated by pions, it is possible to use soft pion methods to obtain it[2, 3, 4, 5]. However to leading order $O(T^2)$ the temperature corrections to correlators may be determined on the basis of current algebra and PCAC without actually using OPE[6, 7, 8]. In this order a mixing of correlators in parity and/or isospin takes place[8, 9]. The important point is that this mixing arises from direct scattering of the heat bath pions on the currents and no thermal mass shift is generated in this order. (A general argument regarding the absence of mass shifts in this order was given in Ref.[6]). It has been shown[3, 10] that the $T$-dependence of the relevant condensates exactly matches the mixing of the correlators leaving no room for mass shifts.

The condensates mentioned above are due to Lorentz scalar operators in the OPE. An important new feature of OPE at $T \neq 0$ is that operators with non-zero Lorentz spin $s$ contribute to correlation functions because their average over the heat bath is non-zero. They contribute to higher orders in $T^2$ and are new non-perturbative parameters to be determined[1, 4, 5]. They cannot be obtained using soft pion approach.

In a recent paper[11] we have shown that order $T^4$ corrections to the vector and axial correlators may be obtained using dispersion relations for the amplitudes of deep inelastic scattering on pions and discussed their relation to the lowest $s = 2$ condensate, the energy-momentum tensor. The results were interpreted in terms of $\rho$ and $a_1$ thermal mass shifts which turned out to be negative. In the present paper we consider $T^4$ corrections for correlators with quantum numbers of a nucleon and isobar. There is no similarity with deep inelastic scattering in this case, but we can use OPE with account of the $s = 2$ operators to obtain the corrections responsible for the mass shifts.

Let us consider the thermal correlation function defined as

$$C_N(q, T) = i \int d^4 x e^{iqx} \langle T\{\eta(x), \bar{\eta}(0)\}\rangle_T \tag{1}$$

where

$$\langle T\{\eta(x), \bar{\eta}(0)\}\rangle_T = \frac{\sum_n \langle n|T\{\eta(x), \bar{\eta}(0)\}e^{-H/T}|n\rangle}{\sum_n \langle n|e^{-H/T}|n\rangle}, \tag{2}$$

Here $H$ is the QCD Hamiltonian, the sum is over all states of the spectrum, and the nucleon (proton) current is taken to be[12]

$$\eta = (u^a C \gamma_\mu u^b) \gamma_5 \gamma_\mu d^c \, \epsilon^{abc} \tag{3}$$

where $C = \gamma_2 \gamma_0$ is the charge conjugation matrix. It is assumed that $q^2$ is space-like, $Q^2 = -q^2 > 0$, and $Q^2$ is much larger than a characteristic hadronic scale, $Q^2 \gg R_c^{-2}$, where $R_c$ is the confinement radius, $R_c^{-1} \sim 0.5$ GeV.

To determine the correlator $C_N$ at low $T$ one has to calculate the matrix elements in Eq.(2) over pions. In the chiral limit the pion momenta are of order $T$, and the leading $O(T^2)$ corrections are obtained neglecting the pion momentum in the one-pion matrix elements. In this order using PCAC, current algebra and doing the integral over the thermal pion phase space

$$\int \frac{d^4 p}{(2\pi)^4} \frac{\pi \delta(p^2)}{e^{|\mathbf{p}|/T} - 1} \sum_a \langle \pi^a(p) | T\{\eta(x), \bar{\eta}(0)\} | \pi^a(p) \rangle \tag{4}$$

gives the exact result[7, 10]

$$C_N(q, T) = (1 - \xi) C_N(q, 0) - \xi \gamma_5 C_N(q, 0) \gamma_5 \tag{5}$$

where

$$\xi = T^2 / 16 F_\pi^2 \tag{6}$$

and $F_\pi = 93$ MeV is the pion decay constant. The $T^2/F_\pi^2$ corrections are due to the contact interactions of pions with the currents (Fig.1, $a$ and $b$). There is no need to use OPE to calculate $C_N$ in this order. Indeed, the $T$-dependences of $s = 0$ condensates which contribute to this order exactly satisfy Eq.(5)[10]. Thus, in order $T^2$ the finite temperature correlator $C_N(q, T)$ is expressed in terms of the $T = 0$ correlators. This is similar to the mixing of the vector and axial correlators[8], but in the nucleon case both $C_N$ and $\gamma_5 C_N \gamma_5$ are contributed by the nucleon and its parity partner $N(1535)$. The choice of the nucleon current in Eq.(3) allows to avoid the compensation of their contributions to the chirality violating part of $C_N$[13]. It is evident from Eq.(5) that there can be no nucleon mass shift in order $T^2$. The absence of the nucleon mass shift in this order was also demonstrated in Ref.[6] by an explicit calculation of the $\pi N$ self-energy correction to the nucleon propagator at $T \neq 0$. The mass shift is expected to appear in the next order, $T^4$.

The $T^4$ corrections to $C_N$ are due either to two-pion matrix elements with zero pion momenta, or to one-pion matrix elements with non-zero pion momentum[11]. The former are of order $T^4/F_\pi^4$. They come from the contact interaction of pions with the currents
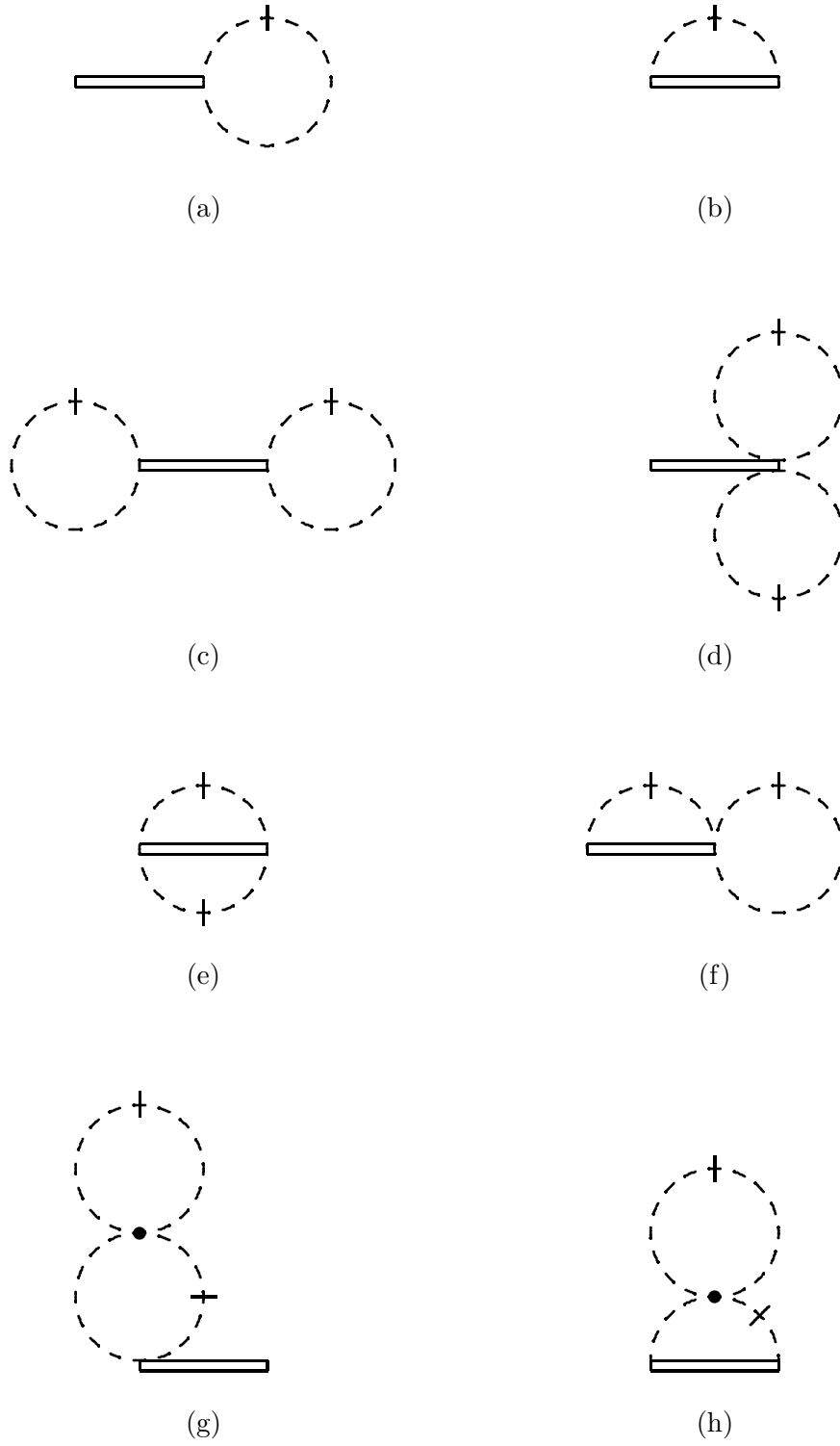
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Fig. 1. Contact terms to two loops. Double lines are $T = 0$ correlators, crossed dashed lines denote thermal pions.

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in two loops (Fig.1, $c$ through $h$), can be calculated without OPE, and contribute to the change in the coupling of the nucleon to the current (see Eq.(13) and (15) below). The latter are of order $T^4/Q^4$, they arise from the pion scattering on the intermediate state in the correlator and may be interpreted in terms of a thermal mass shift. They will also break the Lorentz covariance preserved in order $T^2$ as is seen from Eq.(5) in which $C_N$ has the form

$$C_N(q^2, T) = C_1(q^2, T)\hat{q} + C_2(q^2, T) \tag{7}$$

where $\hat{q} = q_\mu \gamma_\mu$. The general form of the correlator in the rest frame of the heat bath is given by

$$C_N(q, T) = C_1(q, T)\gamma_0 q_0 + C_2'(q, T)\boldsymbol{\gamma q} + C_2(q, T) \tag{8}$$

where $C_1' \neq -C_1$ and all $C$'s depend separately on $q_0$ and $|\mathbf{q}|$. We will consider the case of $\mathbf{q} = 0$, $q_0 \neq 0$, when we are left with the same number of independent structures as at $T = 0$.

In terms of OPE, there are two $s = 2$ operators in the leading twist which provide the $T^4/Q^4$ corrections to the correlator. They are the quark and gluon energy-momentum tensors

$$
\begin{aligned}
\theta_{\mu\nu}^q &= \frac{i}{2}(\bar{q}\gamma_\mu D_\nu q + \bar{q}\gamma_\nu D_\mu q), \quad (q = u, d) \\
\theta_{\mu\nu}^G &= G_{\mu\alpha}^a G_{\alpha\nu}^a - \frac{1}{4}g_{\mu\nu}G_{\alpha\beta}^a G_{\beta\alpha}^a
\end{aligned} \tag{9}
$$

normalized according to

$$\langle \pi(p)|\theta_{\mu\nu}^u + \theta_{\mu\nu}^d + \theta_{\mu\nu}^G|\pi(p)\rangle = 2p_\mu p_\nu \tag{10}$$

where $\langle \pi(p)|\pi(p')\rangle = (2\pi)^3 \, 2E \, \delta^{(3)}(\mathbf{p} - \mathbf{p}')$. The contributions of the quark and gluon tensors to Eq.(10) are about the same[14, 4]. However, the gluon tensor enters OPE for the correlator being multiplied by $\alpha_s(Q^2)$ and we will neglect its contribution. In what follows we will also omit the anomalous dimension of $\theta_{\mu\nu}^q$.

The calculation of the quark tensor contribution to the correlator can most easily be done in the coordinate representation by cutting quark lines and using the second term in the expansion of the quark operator, $q(x) = q(0) + x_\lambda D_\lambda q(0) + ....$

Cutting one quark line gives the chirality conserving part. In case of $d$-quark this results in matrix elements of the form $\langle \pi(p)|\bar{d}D_\lambda d|\pi(p)\rangle$ which are proportional to the

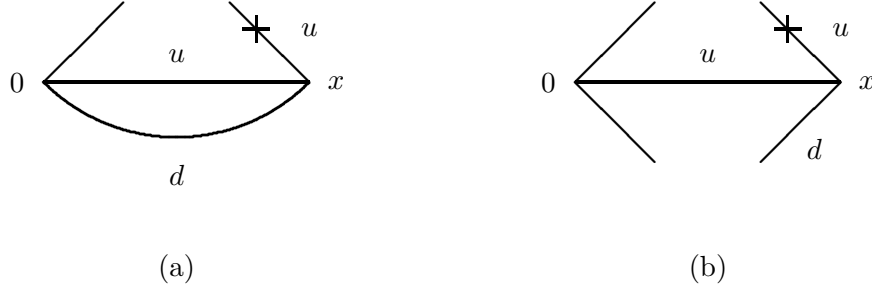4

(a)                                    (b)

Fig. 2. Leading terms in OPE related to the nucleon thermal mass shift. The quark leg with a cross corresponds to $x_\lambda D_\lambda q(0)$.

pion momentum $p$ and vanish under the integration in Eq.(4). Cutting a $u$-quark line (Fig.2a) gives

$$i\frac{48M_2}{\pi^4 x^8}(\hat{p}(px)x^2 + \hat{x}(px)^2) \tag{11}$$

where $M_2$ is defined according to $\langle\pi(p)|\theta^u_{\mu\nu} + \theta^d_{\mu\nu}|\pi(p)\rangle = 8M_2 p_\mu p_\nu$ and is equal to the second moment of the pion structure function $M_2 = \int_0^1 F_2(x)dx$[11].

Integrating over the thermal pion phase space in the rest frame of the heat bath with the help of

$$\int \frac{d^3p}{(2\pi)^3 \, 2|\mathbf{p}|} \frac{p_\mu p_\nu}{\exp(|\mathbf{p}|) - 1} = (-g_{\mu\nu} + 4g_{\mu 0}g_{\nu 0})\frac{\pi^2 T^4}{180} \tag{12}$$

and going over to the momentum representation, we get the expression to order $T^4$ for the chirality conserving structure proportional to $\gamma_0 q_0$

$$C_1(Q^2, T) = C_1(Q^2, 0) - \frac{4M_2}{15} T^4 \ln Q^2 \tag{13}$$

(order $\xi$ and $\xi^2$ contact terms cancel in this structure).

Similarly, the chirality violating part of the $T^4/Q^4$ correction is obtained by cutting two quark lines. Here, cutting the two $u$-quark lines gives terms linear in the pion momentum which vanish under the integration over pions, while cutting the $d$-quark and a $u$-quark line (Fig.2b) gives terms quadratic in the pion momentum

$$-\frac{24M_2(px)^2}{3\pi^2 x^4}\langle\bar{q}q\rangle \tag{14}$$

After integration over pions, this gives in the momentum representation

$$C_2(Q^2, T) = \left(1 - 2\xi - \frac{2}{3}\xi^2\right) C_2(Q^2, 0) - \frac{2\pi^2 M_2 \langle \bar{q}q \rangle}{5} \frac{T^4}{Q^2} \tag{15}$$

Here $\langle \bar{q}q \rangle = -(240 MeV)^3$ is the usual $T = 0$ quark condensate [†]. In Eq.(13) and (15) we have taken into account order $T^4/F_\pi^4$ corrections in a way similar to the case of vector and axial correlators[11]. The full answer was again obtained by taking into account the initial (finite) state interaction between two pions (Fig.2, $g$ and $h$). The leading term in OPE for $C_2(Q^2, 0)$ is proportional to $\langle \bar{q}q \rangle$. A straightforward check shows that the $\xi^2$ term in Eq.(15) agrees with the known[2] $T^4$ correction to the quark condensate. It is easy to see that the last terms in Eqs.(13) and (15) are indeed of order $T^4/Q^4$ compared to $C_1(Q^2, 0)$ and $C_2(Q^2, 0)$, respectively.

Now, we would like to interpret the $T^4$ corrections to the correlators in Eqs.(13) and (15) in terms of the nucleon thermal mass shift. The functions $C_1$ and $C_2$ may be presented in the form of dispersion integrals

$$C_i(Q^2, T) = \frac{1}{\pi} \int_0^\infty \frac{\mathrm{Im} C_i(s, T) ds}{s + Q^2} \tag{16}$$

where a sufficient number of subtractions is implied and a standard model (lowest resonance + continuum) for the spectral densities is usually assumed

$$\mathrm{Im} C_i(s, T) = \pi \lambda_i^2(T) \delta(s - m_N^2(T)) + \theta(s - s_0(T)) \rho(s, T) \tag{17}$$

At $T = 0$ the nucleon pole contribution to the correlator is

$$C_N^{(pole)}(Q^2, 0) = \lambda_N^2 \frac{\gamma_0 q_0 + m_N}{Q^2 + m_N^2} \tag{18}$$

where $\lambda_N = \lambda_1(0) = \lambda_2(0)$ is the coupling of the nucleon to the current,

$$\langle 0|\eta|N\rangle = \lambda_N v \tag{19}$$

and the nucleon spinor $v$ is normalized as $\bar{v}v = 2m_N$. At $T \neq 0$ the general form of the nucleon propagator is

$$\frac{1}{\gamma_0 q_0 (1 + a_N) - m_N (1 + b_N)} \tag{20}$$

[†]In deriving Eq.(14) we used the following approximation for the matrix element over pion $\langle \pi(p)|\bar{u}\gamma_\mu D_\nu u\,\bar{u}u|\pi(p)\rangle = \langle \pi(p)|\bar{u}\gamma_\mu D_\nu u|\pi(p)\rangle\langle\bar{u}u\rangle$. An estimate of the accuracy of this factorization may be obtained by inserting a pion intermediate state. Using the scale of OPE $\mu \sim 0.5$ Gev as the cut-off in the integral over the momentum, we get that this contribution is suppressed as $\mu^2/8\pi^2 F_\pi^2 \approx 0.3$.

where $a_N$ and $b_N$ are $T$ dependent corrections. Multiplying the numerator and the denominator in the above formula by $\gamma_0 q_0 (1 + a_N) + m_N (1 + b_N)$ we identify the thermal shift of the nucleon pole as

$$\delta m_N(T) = m_N(b_N(T) - a_N(T)), \tag{21}$$

While $\delta m_N(T)$ here is of order $T^4$, the functions $a_N(T)$ and $b_N(T)$ contain $T^2$ terms[6], $a_N(T) = g_A^2 \xi + O(T^4)$, $b_N(T) = g_A^2 \xi + O(T^4)$, where $g_A = 1.27$ is the usual axial coupling of the nucleon. They are due to the contribution of the $\pi N$ self-energy graph at $T \neq 0$. On the other hand, Eq.(13) and (15), which are exact, do not contain any terms involving $g_A^2$. This seeming paradox was resolved in Ref.[10] in which it was shown that the $g_A^2$ terms indeed cancel provided the $\pi N \to N$ scattering is accounted for in the spectral density of the correlator at $T \neq 0$. Taking this into consideration we obtain the nucleon contributions to $C_1$ and $C_2$

$$
\begin{aligned}
C_1(Q^2, T) &= \lambda_1^2(0) \frac{1 - \bar{a}_N(T)}{Q^2 + m_N^2 + \delta m_N^2(T)} \\
C_2(Q^2, T) &= \lambda_2^2(0) \left(1 - 2\xi - \frac{2}{3}\xi^2\right) \frac{m_N(1 - \bar{a}_N(T)) + \delta m_N(T)}{Q^2 + m_N^2 + \delta m_N^2(T)}
\end{aligned}
\tag{22}
$$

where $\bar{a}_N(T) = O(T^4)$ is the nucleon wave-function renormalization due to $\pi X$ self-energy corrections to the nucleon propagator at $T \neq 0$, where $X \neq N$. To relate $\delta m_N(T)$ and $\bar{a}_N(T)$ to the $T^4$ corrections obtained before, we match Eq.(22) against Eq.(13) and (15) keeping terms linear in $\delta m_N$ and $\bar{a}_N$. To suppress the contribution of higher states, the Borel transformation

$$\hat{B}C(Q^2) = \lim_{\substack{Q^2, n \to \infty \\ Q^2/n = M^2 = const}} \frac{1}{(n-1)!}(Q^2)^n \left(-\frac{d}{dQ^2}\right)^n C(Q^2) \tag{23}$$

is usually performed in QCD sum rules. Though we neglect the changes in the continuum induced by temperature, the Borel transformation is still necessary to kill the subtraction terms proportional to $T^4$ in Eq.(13). Applying it, we get

$$
\begin{aligned}
\frac{\delta m_N}{m_N} &= -\frac{4M_2 M^2 e^{m_N^2/M^2}}{15\lambda_N^2}\left(1 + \frac{3\pi^2 \langle \bar{q}q \rangle}{2M^2 m_N}\right) T^4 \\
\bar{a}_N &= -\frac{4M_2 M^2 e^{m_N^2/M^2}}{15\lambda_N^2}\left(1 - \frac{2m_N^2}{M^2} - \frac{3\pi^2 m_N \langle \bar{q}q \rangle}{M^4}\right) T^4
\end{aligned}
\tag{24}
$$

We see that the mass shift is negative. If $M^2$ varies between 0.7 and 2 MeV$^2$ (the stability range of the $T = 0$ sum rules for the nucleon), $\delta m_N$ changes by 30%, while $\bar{a}$ is less

stable. Taking $M = m_N$, $M_2 = 0.12$ ‡ and $\lambda_N^2 = 1.2 \cdot 10^{-3}$ GeV$^6$[12], we find that the mass shift is rather small: $\delta m_N \approx -2$ MeV at $T = 80$ MeV. We note that up till this temperature our estimate for $\delta m_N$ agrees with the result of Ref.[6], obtained using experimental information on the $\pi N$ forward scattering amplitude.

Taking into account higher twist $s = 2$ condensates, when they become available, one could make the above predictions more accurate. (Note, however, that $\lambda_N^2$ is known with an accuracy of 50%[12]). Also, to go to higher $T$ in this approach it is necessary to known higher spin condensates which contribute to next terms in the low $T$ expansion of $\delta m_N$.

We have also considered the case of $\Delta$-isobar. In that case the current in question is[12]

$$\eta_\mu = (u^a C \gamma_\mu u^b) u^c \, \epsilon^{abc} \tag{25}$$

The corresponding correlator $C_{\mu\nu}^\Delta(q)$ has a number of tensor structures already at $T = 0$ and is contributed not only by $\Delta$, but also by the resonance with $J^P = \frac{1}{2}^-$. However, the structures $g_{\mu\nu}\hat{q}$ and $g_{\mu\nu}$ correspond only to $J^P = \frac{3}{2}$ states. At $T \neq 0$ independent structures proliferate and we again simplify the situation by putting $\mathbf{q} = 0$. The calculation is quite similar to the case of nucleon. The pion matrix elements which give rise to the $T^4/Q^4$ corrections proportional to $g_{\mu\nu}$ are

$$- i \frac{24 M_2}{\pi^4 x^8} ((p_\mu x_\nu + p_\nu x_\mu) \hat{x}(px) - g_{\mu\nu} \hat{x}(px)^2) \tag{26}$$

for the chirality conserving and

$$\frac{4 M_2 \langle \bar{q}q \rangle}{\pi^2 x^4} ((p_\mu x_\nu + p_\nu x_\mu)(px) - g_{\mu\nu}(px)^2) \tag{27}$$

for the chirality violating structure. Integrating over the thermal pion phase space and going to the momentum representation we get

$$\frac{2 M_2}{45} T^4 \ln Q^2 \; \cdot g_{\mu\nu} q_0 \gamma_0 \tag{28}$$

and

$$\frac{8 M_2 \langle \bar{q}q \rangle}{45} \frac{T^4}{Q^2} \; \cdot g_{\mu\nu} \tag{29}$$

Matching these corrections against the contribution of $\Delta$ to the spectral density, we get after the Borel transformation

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‡See [11] for a discussion of existing estimates of $M_2$.

$$\frac{\delta m_\Delta}{m_\Delta} = -\frac{2M_2 M^2 e^{m_\Delta^2/M^2}}{45\lambda_\Delta^2}\left(1 + \frac{4\pi^2\langle\bar{q}q\rangle}{M^2 m_\Delta}\right)T^4$$

$$\bar{a}_\Delta = -\frac{2M_2 M^2 e^{m_\Delta^2/M^2}}{15\lambda_\Delta^2}\left(1 - \frac{2m_\Delta^2}{M^2} - \frac{8\pi^2 m_\Delta\langle\bar{q}q\rangle}{M^4}\right)T^4 \tag{30}$$

The mass shift is again negative. Substituting $\lambda_\Delta^2 = 2.5 \cdot 10^{-3}$ GeV$^6$[12] and putting $M = m_\Delta$, we see that mass shift is much smaller than the nucleon one: $\delta m_\Delta \approx -0.25$ MeV at $T = 80$ MeV.

To summarize, we have used the only presently available non-scalar condensate of spin 2, the energy-momentum tensor, to calculate full order $T^4$ corrections to the correlators with quantum numbers of nucleon and $\Delta$-isobar. Expressed in terms of physical nucleon and $\Delta$ contributions, these corrections determine the leading low $T$ behavior of the thermal mass shifts, which are of order $T^4$ and negative, as in the case of $\rho$ and $a_1$ mesons[11]. The mass shifts are rather small numerically, reaching a few MeV at $T \sim 100$ MeV. This is an order of magnitude less than what one would get from the naive scaling, $m_{N(\Delta)}(T) \sim (-\langle\bar{q}q\rangle_T)^{1/3}$, motivated by the results of Ref.[12].

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